Indian Statistical Institute
II Sem, Mid-Semestral Examination 2008-2009
B.Math.(Hons). II year
Analysis IV

Date:02-03-2009 Duration: 3 Hours Instructor: B.Bagchi

Max Marks 100

1. Prove from definitions:

   (a) Every compact metric space is complete,

   (b)Every totally bounded complete metric space is compact.

   $[10 + 15 = 25]$

2. If X is a complete metric space and $T : X \longrightarrow X$ is a function such that for some constant $c, 0 < c < 1$, we have $d(Tx, Ty) \leq c.d(x, y)$ for all $x, y \in X$ then show that T has a fixed point. [20]

3. If X is a compact metric space and $T : X \longrightarrow X$ is an isometry then show that T is onto. Give an example to show that if X is not compact then this need not be true. [25]

4. If X is a metric space and $C_b(X)$ is the metric space of all bounded continuous functions from X to $\mathbb{R}$ with the sup norm, then show that

   (a) $C_b(X)$ is complete.

   (b) X embeds isometrically in $C_b(X)$.

   (c) If $C_b(X)$ is a finite dimensional vector space then X is finite.

   $[10 + 10 + 10 = 30]$